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Point Allocation, version 2.0

Introduction

Point Allocation is a method designed to allocate the points an NHL team earns amongst its players, in proportion to the amount each player on the team contributed to earning these points. It is a comprehensive measure of player value, encompassing all aspects of the game. When I first published it, Point Allocation was state-of-the-art (such was the state of the art), but that was well over two years ago, and things have changed.

This leads to the reasons for developing an updated version of the method. For one, others have created methods to measure the same thing as Point Allocation, in one form or another. Tom Awad's Goals Versus Average is at least as good as my original method (and is probably better); Alan Ryder's Player Contribution is clearly superior. I needed to get my method up to snuff. Also, upon rereading my original work, in addition to a significant error in one formula which several people had pointed out, I found that I woefully under-explained Point Allocation. No worries of that here; I'll be as detailed as possible, or at least desirable.

Point Allocation features a healthy mix of art and science. Many generally-useful hockey stats are still quite fuzzy. That is, some statitics (such as the component parts of plus-minus) do contain some amount of information about the individual player, but there is also a good deal of 'noise' produced by the player's environment. The methods of Ryder and Awad are more mathematical in nature than mine. But I believe that due to the paucity of statistics in hockey, some artistry is called for.

Marginal Goals

We start once again with Marginal Goals, an idea adapted from Bill James' concept of Marginal Runs, from his (much-criticized, often unjustly so) book *Win Shares*. Marginal Goals represent what even the poorest major-league team of all time would do in competition. Even the worst NHL team will score a few goals, and won't allow an infinite number of them. Marginal Goals represent what you can achieve without really trying. A team's Marginal Goals For (MGF) and Marginal Goals Saved (MGS) represent what a team achieves above that minimum level, and are defined as follows:

MGF = GF - (AvgG x 0.6)

MGS = (AvgG x 1.6) - GA

Where AvgG is the league average team goals-for (or goals-against) total, GF is the team's goals for, and GA is the team's goals against.

From there, you can produce a very accurate estimation of a team's winning percentage, and therefore expected points (EPTS), as follows:

EPTS = ((MGF + MGS) / (2 x AvgG)) x 2 x SCH

Where SCH is the length of schedule (currently 82).

Ryder demonstrated that this estimate is only ever-so-slightly less accurate than the more-common, and widely-accepted Pythagorean projection method. What Marginal Goals allows us to do (but Pythagorus or any of its variants does not) is allocate the EPTS between offence (MGF) and defence (MGS). Offensive Points (OP) and Defensive Points (DP) are allocated as follows:

$$OP = MGF / (MGF + MGS) \times EPTS$$

$$DP = MGS / (MGF + MGS) \times EPTS$$

You may notice, if you work through the math, that OP will make up 40% of a league's points, while DP will make up 60% of the total. This is a departure from my original work, which used a 50/50 split between the two. So why the change? Well, Bill James uses a 48/52 split for baseball's Win Shares, though his justification is weak; and Ryder uses a 42/58 split in his method, which is very similar to the split used here, though his justification is also fairly weak.

So Why 40/60, Then?

We begin with the realization that the offence/defence split question is inextricably linked with the theory of Marginal Goals. A Marginal Goal Rate (MGR) of 0.6 (i.e., using 0.6 to calculate MGF and 1.6 for MGS) will produce a 60% allocation to defence. An MGR of 0.5 will produce a defensive allocation of 50%. An MGR of 0.55 will produce a defensive allocation of 55%. And so on.

But does the relationship hold going the other way? Could you not, for instance, produce a 40/60 split by using 0.8 and 1.3 to calculate Marginal Goals? Yes, you could. But your winning percentage estimates would be way off; try it and see. In order for the predictive element of Marginal Goals to work, the coefficients must be in the form (K, 1+K), where K is between 0 and 1. Note that the estimates would work for any value of K except 0; the limits presented are conceptual in nature. Anything negative, or greater than 1 would produce really funky allocations between offence and defence.

This means that if we can arrive at a good estimate for K based on actual data, we will be able to estimate what percentage of hockey offence and defence represent. But what data to use? Let's look again at what Marginal Goals represents.

An MGR of 0.6 basically means that no matter how bad a team you have, you'll still score at least 60% of the league average in goals, and allow no more than 160% of the league average in goals against. These are the effective boundaries of team performance, which exist because the NHL is the highest level of competition in hockey, and players are screened extensively before getting a chance to play there. It seems to me, then, that if we examine the worst teams in NHL history, we can get an indication of what K is, and thus what the MGR should be.

So here are the 18 worst NHL offenses of all time, as determined by GF divided by AvgG. Why 18? Why not 18?

Chicago	1928/29	0.51
Mtl. Maroons	1924/25	0.60
Chicago	1938/39	0.64
Boston	1924/25	0.65
Washington	1974/75	0.66
Ottawa	1992/93	0.66
Kansas City	1974/75	0.67
NY Islanders	1972/73	0.67
Kansas City	1975/76	0.70
Tampa Bay	1997/98	0.70
St. Louis	1934/35	0.71
Minnesota	1975/76	0.71

Pittsburgh	1928/29	0.72
NY Americans	1930/31	0.72
Philadelphia	1930/31	0.72
Chicago	1931/32	0.72
San Jose	1992/93	0.72
NY Islanders	1973/74	0.73

And here are the 18 worst NHL defences of all time, as determined by GA divided by AvgG:

Philadelphia	1930/31	1.75
Washington	1974/75	1.63
Chicago	1927/28	1.60
Boston	1924/25	1.59
Quebec	1919/20	1.54
NY Americans	1927/28	1.53
NY Rangers	1943/44	1.52
Chicago	1950/51	1.48
Ottawa	1993/94	1.46
NY Americans	1940/41	1.45
Boston	1961/62	1.45
Chicago	1946/47	1.44
Chicago	1953/54	1.44
Washington	1975/76	1.44
Pittsburgh	1929/30	1.42
Hamilton	1922/23	1.41
Mtl. Canadiens	1939/40	1.40
NY Rangers	1942/43	1.40

By themselves, these lists are interesting, but...well, what if we combine them? Suppose we take the worst offence, along with the MGR implied, and pair it with the worst defence, along with the MGR implied there. Then we can take the average of the implied MGR's, and do the same for the second-worst pair, then the third-worst, etc.:

Pair	MGR
1	0.63
2	0.62
3	0.62
4	0.62
5	0.60
6	0.60
7	0.60
8	0.58
9	0.58
10	0.58
11	0.58
11	0.58
12	0.58
13	0.58
14	0.58
15	0.57
16	0.57
17	0.56
18	0.57

Historically bad teams, then, seem to indicate an MGR of about 0.62. These teams would be as bad as you can get in major-league hockey, and are representative of the crux of Marginal Goal analysis. That's why we use an MGR of 0.6. It's rounded off for simplicity, and because using a number like 0.62 would be pretending to have more precision than we really have here. This analysis suggests that hockey is 40% offence and 60% defence.

Which Points to Allocate?

Most player value systems (including the original Point Allocation) allocate a team's actual points (or what have you) amongst its players. But the basis of this new system is Marginal Goals analysis, which produces an estimate of the number of points a team should have based on a team's goals for and goals against. Therefore, we use a team's EPTS rather than actual points.

Why? There is no evidence that the differences between actual and expected points are the result of anything other than luck and random chance. Why should we evaluate players based on luck? Chances are, if a particular season were replayed (which, of course, is not possible), any differences between actual and expected points would even out.

The use of EPTS also avoids a sticky situation. Using actual points, teams would have varying rates of Marginal Goals to points. This would imply that some teams are more "efficient" in their use of Marginal Goals, which in many ways makes no sense whatsoever, and at any rate is contrary to Marginal Goals analysis, wherein each Marginal Goal is given equal value when calculating EPTS.

A Note on Replacement Value

Bill James' Win Shares is often criticized for lacking "Loss Shares"; similarly, Point Allocation has been criticized for lacking a component involving negative contribution (not that that concept makes any sense). However, another of my essays establishes a theoretical framework in which not only are "Loss Shares" irrelevant, they simply do not exist.

But replacement value is still a valid concern. Point Allocation does not incorporate replacement value in its calculations, and this must be borne in mind when using it. In due time I will detail the Points Above Replacement method (PAR), which will combine Point Allocation with a replacement-value element. But for now, let us continue with Point Allocation.

Allocating Offensive Points

Having divided points between offence and defence, we now proceed to the process of allocating these points to individual players. We'll start with offence, for the simple reason that it's easier and more straight-forward then defence.

First, we realize that it doesn't matter when or how a goal is scored, it counts as precisely one goal. Therefore, in terms of offence, goals scored at even strength (ES), while short-handed (SH) or on the power-play (PP) are all equivalent in value. We need concern ourselves only with a player's scoring totals; situational breakdowns, while interesting, are irrelevant here (though strangely enough, we'll use them later to evaluate a player's defence).

Now we need to make an assumption. What is the relative value of an assist to a goal? Are they of equal value, as the Art Ross Trophy assumes? I'm not sure. True, there are many goals where the playmakers are the key players, and the goal-scorer was merely "johnny-on-the-spot". But more frequently, I think, there are goals on which "touch" assists are recorded, with the assister having little to do with the scoring play, but having touched the puck for a second before the goal was scored. Therefore, I think each goal should be somewhat more valuable than each assist.

An easy way to achieve this is to assume that goal-scoring represents half of the value of scoring, and playmaking represents the other half. Since there are typically 1.7 assists per goal in the modern NHL, this works out to each 'goal' being worth 0.50 of a goal scored, with each 'assist' being worth 0.29 of a goal scored (0.50 divided by 1.70).

Two notes here: the value of an assist is calculated for each team depending in the assist/goal ratio of that team, and goaltenders are excluded. Goalies do not receive any credit for offence, because virtually all goaltender assists are of the skill-less, 'touch-only' variety.

To allocate a team's OP, simply add up the Offensive Contribution (OC) of each player (0.50 times goals plus 0.29 times assists) on the team, and distribute the OP based on each player's contribution to this total.

But hold on a second, aren't we using “marginal” goals? Why are we looking only at total goals here then? This is because, unlike in baseball, some hockey players actually contribute nothing (or efffectively nothing) to offence. They are usually stay-at-home defencemen...you will occasionally see a player who plays a full schedule and has something like a goal and three assists.

That is to say, while at the team level there is a certain level of goals (the boundary of Marginal Goals) you expect even the worst team to score, this simply isn't true of individual skaters. For some players their role calls for no offensive contribution, and we will not penalize them for this; they will already receive

next-to-nothing for OC. Here we are overcoming a common flaw in sports analysis: we realize that something that applies on a team level does not necessarily apply on an individual level.
This does, however, lead to a significant problem. In effect, we are using different standards for teams and individual players. But if we make no adjustment for this at the individual level, we end up with players on low-scoring teams receiving fewer OP per OC than players on higher-scoring teams, due to a greater proportion of the goals being "eaten up" by the marginal scoring level. This is undesirable; but how does one avoid it?

To reconcile the differing standards used at the team and individual levels, we introduce the concept of *synergy*. Synergy represents the fact that hockey is a team game, probably moreso than any other team game, if you know what I mean. If you assemble a hockey team, each player of which is exactly 10% above replacement level, your team will not finish with a record that is also 10% better than replacement level. It will be something more like 15-20% better than replacement; I don't know exactly, but it will be more than 10%.

A team's Offensive Synergy (OS) is defined as the OP you would expect a team to have based on the total of all the team's players' OC figures, minus the team's actual OP. Thus a team with an above-average OP will have a positive OS, while a team with a below-average OP will have a negative OS.

OS is effectively another player to allocate OP to; but a purely hypothetical one. Sufficient OP are allocated to OS (either positive or negative) before OP are allocated to individual players, so that each actual player in the league receives a constant amount of OP per OC. OS is the difference between the team's OP as calculated previously (using MGF) and the OP that results from using only GF for the calculation. Since offence accounts for 40% of all points, the formula is:

$$OS = ((SCH \times .4)/(AvgG \times .4) \times MGF) - ((SCH \times .4)/AvgG) \times GF)$$

For example, in 2002/03 Anaheim scored 203 GF, when the league average was 217.7. Anaheim's OS is therefore -3.3, which means we figure they lost 3.3 OP for their below-average offensive synergy. The team's individual skaters therefore do not share in 27.3 OP (calculated using MGF), but 30.6 as follows:

$$OP2 = OP - OS$$

Allocating DP Between Skaters and Goaltenders

We now enter the world of defence in hockey, which, as you may know, is not well-measured statistically. We must therefore proceed based on logic, inference and reasonable assumption.

We'll start with some assumption. We'll assume that each position (the three forwards positions, two defence positions and the goaltender) is of equal value to a team, on average. It's hard to say how reflective of reality this is, but it has a nice element of equitability to it. So, we assume each position typically accounts for one-sixth, or 16.7%, of a team's total value. One position is easy. Goaltenders do not play offence. Therefore the entirety of their value is defensive in nature. 16.7% total value divided by 60% of the game being defence equals goaltenders being responsible for 27.8% of a team's defence, on average. This leaves 72.2% of a typical team's defence (as well as 100% of offence) to allocate to skaters.

Goals allowed are the result of two factors, mathematically speaking: the number of shots allowed, and the proportion of shots stopped. And since goals allowed are the result of these two factors, so are Marginal Goals Saved. We could assign the responsibility of shots allowed to skaters, and the responsibility of shots stopped to goaltenders. But moving from mathematics to reality, we realize that goals against also result from another factor: shot quality. A team's defence may allow only 10 shots in a game, but if all of these shots are point-blank, the goaltender will have his save percentage suffer. The goalie would be unfairly penalized by assigning the full responsibility of save percentage to the one position.

Fortunately, someone has finally done a rigorous analysis on shot quality. The aforementioned Alan Ryder's work is seminole, and is a must-read for any hockey analyst. Ryder's work is incoporated into Point Allocation in order to produce more realistic estimates of goaltender contributions to defence.

I will not take the time to explain Ryder's methods here; that's what the link is for. Suffice it to say that the save percentage used herein is modified by the estimated quality of the shots faced by the goaltender, per Ryder's work.

To be fair to goaltenders, we exclude empty-net goals when calculating goalies' MGS. By considering only non-empty-net shots and goals, we can divide MGS between goaltenders (MGSG) and skaters (MGSS) as follows, to ensure goalies receive 27.8% of the defensive credit:

MGSG = (AvgG x 1.139) - (AvgSA x (1 - SPCT))

MGSS = (AvgG x 1.361) - (SA x (AvgSPCT))

Where AvgSA is the league-average shots against, SPCT is team save percentage (excluding empty-net shots and goals, modified by estimated shot quality), SA is team shots against, and AvgSPCT is league-average save percentage.

But since we want to assign the responsibility of empty-net goals to skaters, we use the following formulae instead (this also gives us an easy way to deal with rounding discrepancies):

MGSG = (AvgG x 1.139) - (AvgSA x (1 - SPCT))

MGSS = MGS - MGSG

Defensive Points are then allocated based on the ratios of MGSG and MGSS to MGS:

DPG = DP x (MGSG / MGS)

DPS = DP x (MGSS / MGS)

Where DPG is DP allocated to goaltenders and DPS is DP allocated to skaters.

Allocating DPG to Individual Goaltenders

Allocating DPG to a team's individual goaltenders is simply a matter of repeating the above process for each goaltender on a team, and allocating DPG based on the total MGS that results.

That is, for each goaltender, calculate MGSIG (MGS - individual goaltender) as follows:

MGSIG = ((AvgG / AvgMIN) x MIN x 1.139) - ((AvgSA / AvgMIN) x MIN x SPCTG)

Where AvgMIN is the league-average team goaltender minutes played, MIN is the goaltender's minutes played, and SPCTG is the goaltender's save percentage (modified by estimated shot quality).

Defensive Points are then allocated to each goaltender (DPIG) as follows:

DPIG = (MGSIG / sumMGSIG) x DPG

Where sumMGSIG is the sum of the team's MGSIG.

Note that sumMGSIG should equal the team's MGSG, but there may be rounding discrepancies, so sumMGSIG is used. In fact, to prevent these rounding discrepancies, sumMGSIG is used instead of MGSG to calculate a team’s allocation between skaters and goaltenders.

Adjusting DPS for Penalties

Another improvement over the original Point Allocation, this adjustment recognizes the fact that players who take (non-coincident) penalties cost their team goals. There is no evidence that some players have the ability to draw penalties, thereby gaining their team goals (it may be true, but there is currently no evidence). So we only consider the “taking penalties” side of the equation.

So how much does a penalty cost? Well, we must bear in mind our basis in marginality. First, we will consider all penalties to be marginal, since some players take no penalties. Second, the cost in goals of a penalty is the goals against over and above the goals that would have been allowed had there been no penalties. Teams allow a certain number of goals while not short-handed; the cost of penalties is only the additional goals created by the resulting short-handed situations.

Fortunately, this calculation is quite easy. First, we estimate league power-play minutes (PPML) as follows:

PPML = (PPOL - PPGL) x 2

Where PPOL is league PP opportunities, and PPGL is league PP goals. For all it simplicity, this formula produces a very accurate estimate at the team level.

Non-PP minutes (NPPML) are therefore:

NPPML = TML - PPML

Where TML is total league minutes.

The cost of penalties in goals is therefore:

CPLG = PPGL - (NPPGL / NPPML x PPML)

Where NPPGL is league non-PP goals.

To translate this cost into points, we need to know how many points a goal is worth. We therefore calculate the number of goals (or Marginal Goals) per expected point earned in the league (GPPT):

GPPT = GL / (TMS x SCH)

Where GL is total league goals (which is equal to total Marginal Goals), and TMS is the number of teams in the league. This calculation is such to avoid any silly things such as awarding a point for losing in overtime.

The cost of penalties, in points, is:

CPLP = CPLG / GPPT

Therefore an individual non-coincident penalty costs:

CP = CPLP / PPOL

Let's run through the calculations for the 2002/03 NHL season, for clarity. The data needed is:

PPOL = 10,876
PPGL = 1,787
TML = 150,016
NPPGL = 4,743
GL = 6,530
TMS = 30
SCH = 82

Running through the formulae, we get:

PPML = 18,178
NPPML = 131,838
CPLG = 1,133
GPPT = 2.65
CPLP = 427.5
CP = 0.039

This means that each penalty costs 0.039 points. This allows us to calculate the cost of penalties for a team as well, which is what we're really after:

CPT = TSH x CP

Where TSH is the team's times short-handed.

CPT is, of course, a negative. Where do these negative points come from? Well, unlike the defensive aspect of Linear Runs, not out of thin air (gratuitous cheap shot). We've already divided all points between offence and defence. To keep the balance, we must add back an amount equal to the team's DPS, resulting in the new factor DPS2:

DPS2 = DPS + CPT

We do this because a team's defensive rating is reduced by taking penalties, and allowing goals. DPS2 reflects what the team's defence would be if they had not taken any penalties.

Notice that we charge each team an equal number of points for each penalty, regardless of their penalty-killing prowess. This is so we do not charge a player more for a penalty simply because his teammates are not good at killing penalties. A team's penalty-killing ability will be reflected in its DPS2.

Allocating CPT to Individual Players

This should simply be a matter of taking each player's non-coincident penalties (i.e., those that cause a short-handed situation) and applying a point cost at the appropriate rate (0.039 points per penalty in 2002/03). The problem is, this data does not exist. Actually, that's not true. The data exists, it just isn't officially compiled by anyone, to my knowledge.

At any rate, we need something to stand in for this data. The number of minor penalties a player takes is a good approximation of non-coincident penalties, in terms of the ratio of total penalties on the team. CPT is allocated to individual players (including goaltenders) based on each player's proportion of the team's total minor penalties.

Defensive Synergy

Before we can proceed further, we must consider consistency. We previously adjusted offence using the concept of synergy. It would be inconsistent to not do the same for defence. Unfortunately, we have nothing to base defensive synergy (DS) on, as we did for OS. However, since the effect of OS is simply to lessen the variation of OP, we can do the same for DS and DP.

Therefore we calculate DS so that it serves to reduce the variation of DPS. But by how much should the variation be reduced? Again, for consistency, we can reflect the OS calculation.

So let’s gp back to the formula for OS. First, we can simplify it:

OS = SCH x .6 x ((GF / AvgG) - 1))

We need to determine a rate from this. We can calculate the rate of OS per (GF - AvgG), which is the difference between the team’s GF and the league average GF. Since SCH and AvgG are constants for any one season, this rate for each season is easilty determined. For 2002/03, the synergy rate for goals (SRateG) is as follows:

SRateG = (G- AvgG) / (SCH x .6 x ((GF / AvgG) - 1)))

Plugging in 82 for SCH and 217.7 for AvgG, we arrive at:

SRateG = 4.42

This is transgormed into the rate for points (SRateP) by dividing it by the goals per point rate we calculated earlier.

SRateP = SRateG x GPPT

SRateP = 4.42 x .377

SRateP = .17

This means that for each point of DPS2 a team is above or below average, we consider .17 of a point to be due to synergy. Thus:

DS = (AvgDPS2 - DPS2) / SRateP

Where AvgDPS2 is the league average DPS2.

The DPS to allocate to each team's skaters is therefore:

DPS3 = DPS2 - DS

Allocating DPS3 to Defensive Situations

Before we allocate DPS3 to individual skaters, we first divide it up into three defensive situations: even-strength, power-play and short-handed. We do this because we will be allocating points to skaters based on their play in each situation. This is a significant improvement over the original version of Point Allocation, wherein DPS were allocated to skaters based on a single factor.

We have previously noted that TML for 2002/03 was 150,016 and PPML was 18,178. This means an average NHL team played 5,001 minutes, including 606 on the power-play and (since a PP minute on one side is a SH minute on the other) 606 short-handed. So, should we allocate 12.1% of DPS3 to SH situations, 12.1% to PP situations and the remaining 75.8% to ES? Of course not. Defence on the PP is a secondary concern. Conversely, SH situations are all about defence, with offence being an afterthought. Clearly far greater DPS3 should be allocated to SH situations than to PP situations, which reflects the fact that far more OP are achieved in PP situations than in SH situations.

But how much more DPS3? Well, look at this data for the 2002/03 NHL season:

GL = 6,530
PPGL = 1,787
SHGL = 230

The only new term is SHGL, which is total league SH goals scored. On the offensive side of the Point Allocation system, then, points were allocated about 69.1% to ES situations, 27.4% to PP and 3.5% to SH. Thus is makes sense, and is consistent, to allocate DPS3 as follows (as a base): 69.1% to ES, 3.5% to PP, and 27.4% to SH situations. The base DPS3 allocations are calculated as follows:

DPS3PP = SHGL / GL

DPS3SH = PPGL / GL

DPS3ES = 1 - DPS3PP - DPS3SH

The DPS3ES calculation is in this form so that any rounding discrepancies will just be lumped into it.

But these are no good for individual teams. We have to adjust for two things: the amount of time a team spends in each situation, and the team's effectiveness in each situation.

The adjustment for time is easy, using our earlier methods to estimate time in each situation. The effectiveness adjustment is more complex, due to goaltending. Teams with good goaltending will tend to have good penalty-killing as well. To avoid rewarding skaters for good goaltending, we compare performance in PP and SH situations to ES situations, and compare this comparison to the league average for such comparisons. So much for clarity; let's keep going to explain better.

For instance, in the 2002/03 season, the average NHL team allowed 150 ES goals, 60 PP goals and 8 SH goals, in 3,789 ES minutes, 606 SH minutes and 606 PP minutes. The baseline is therefore 0.040 goals allowed per ES minute. The average SH rate is 0.099 per minute, or 2.48 times the rate at ES. The average PP rate is 0.013 per minute, or 0.33 times the rate at ES.

Thus, any team whose SH rate is over 2.48 times their ES rate will have their DPS2SH reduced accordingly, since they are less effective at killing penalties than one would expect. The converse is also true, and the relationship is the same for DPS2PP.

Therefore the team DPS3 by-situation figures (DPS3EST, DPS3PPT, DPS3SHT) are as follows, factoring the effectiveness adjustment as well as the time adjustment:

$$\mathrm{DPS3PPT} = \mathrm{DPS3} \times \mathrm{DPS3PP} \times (\mathrm{PPMT} / (\mathrm{PPML} / \mathrm{TMS})) \times ((((\mathrm{PPGL} / \mathrm{TMS}) / (\mathrm{PPML} / \mathrm{TMS})) / ((\mathrm{ESGL} / \mathrm{TMS}) / (\mathrm{ESML} / \mathrm{TMS}))) / ((\mathrm{PPGAT} / \mathrm{SHMT}) / (\mathrm{ESGAT} / \mathrm{ESMT})))$$

$$\mathrm{DPS3SHT} = \mathrm{DPS3} \times \mathrm{DPS3SH} \times (\mathrm{SHMT} / (\mathrm{PPML} / \mathrm{TMS})) \times ((((\mathrm{SHGL} / \mathrm{TMS}) / (\mathrm{PPML} / \mathrm{TMS})) / ((\mathrm{ESGL} / \mathrm{TMS}) / (\mathrm{ESML} / \mathrm{TMS}))) / ((\mathrm{SHGAT} / \mathrm{PPMT}) / (\mathrm{ESGAT} / \mathrm{ESMT})))$$

$$\mathrm{DPS3EST} = \mathrm{DPS3} - \mathrm{DPS3PPT} - \mathrm{DPS3SHT}$$

Where PPMT is team PP minutes, ESGL is total league ES goals, ESML is total league ES minutes, PPGAT is team PP goals against, SMHT is team SH minutes, ESGAT is team ES goals against, ESMT is team ES minutes, amd SHGAT is team SH goals against.

Allocating DPS2PPT, DPS2SHT and DPS2EST to Individual Skaters

DPS3PPT is allocated very simply. A player's PPM is compared to the sum of all the team's players' PPM, and DPS3PPT are allocated based on the same ratio. We do this because it is difficult to separate the defensive performance of individual players in this situations, and defence is not of prime importance on the PP.

DPS3SHT is somewhat more complex. Since defence is of prime importance when short-handed, it is appropriate to make an adjustment to SHM before allocating DPS3SHT based on it. Since ice time is limited by such things as short shifts and line changes, a player's SHM does not truly reflect a player's ability while short-handed. The best penalty-killer on a team does not receive as much ice time in that situation as he would if defensive ability were the only consideration. Therefore we adjust SHM to increase the variation in SHM between players. The formulae for forwards (SHM2F) and defencemen (SHM2D) are:

$$\mathrm{SHM2F} = ((((\mathrm{SHM} / ((\mathrm{TSHMF} / \mathrm{TGPF}) \times \mathrm{GP})) - 1) \times 1.5) + 1) \times ((\mathrm{TSHMF} / \mathrm{TGPF}) \times \mathrm{GP})$$

$$\mathrm{SHM2D} = ((((\mathrm{SHM} / ((\mathrm{TSHMD} / \mathrm{TGPD}) \times \mathrm{GP})) - 1) \times 1.5) + 1) \times ((\mathrm{TSHMF} / \mathrm{TGPF}) \times \mathrm{GP})$$

Where SHM is a player's SH minutes, TSHMF is the total SHM for a team's forwards and TSHMD is the total SHM for a team's defencemen.

The adjustments can produce SHM2 figures than are less than zero. This makes no sense; therefore, any negative SHM2 is set at zero.

DPS3SHT are then allocated to individual players based on each player's proportion of the team total SHM2.

Much more complex is the method for allocating DPS3EST. PP and SH situations are one-dimensional. If you're on the PP, you're there because of your offence (the majority of the time). Players who kill penalties are doing it because of their defence. But at ES, players must play offence and defence. The difficulty lies in evaluating the defence. Offence is well-measured; how do we handle defence?

In the original version of Point Allocation, I introduced a system which Alan Ryder has since dubbed the "Fyffe Method". The essence of the method is this: playing time should approximate the sum of offensive contribution and defensive contribution. Since we know a player's playing time, and have a good estimate of offensive contribution, we should be able to arrive at a good estimate of defensive contribution. For the new version, this basic method is heavily refined and modified.

In determining a player's ice time, a coach must weigh each player's offensive and defensive abilities, in relation to the other players on the team. This is key: a player competes only against his teammates for ice time; the rest of the league is irrelevent.

We start with offence; even-strength offence, that is. To help illustrate the process, I'll use the 2001/02 Sharks. Teemu Selanne, for instance, had an OC of 12.98 at ES; we'll call this his OCES. He played 82 games and 994 ESM. The numbers for Mike Ricci are 13.25, 79 and 946. We do these calculations for all forwards on the Sharks, for a total OCES (TOCESF) of 135.63 in 972 GP (TGPF) and 10,726 total ESM (TESMF). From there we can figure each forward's Offensive ESM (OFFESMF), which represents what a player's playing time might be if offence were the only consideration, as follows:

$$\mathrm{OFFESMF} = (\mathrm{OCES} / \mathrm{ESM}) / (\mathrm{TOCESF} / \mathrm{TESMF}) \times (\mathrm{TESMF} / \mathrm{TGPF}) \times \mathrm{GP}$$

Now, since we consider hockey to be 40% offence and 60% defence, we can derive a player's Defensive ESM (DEFESMF) as follows:

$$\mathrm{DEFESMF} = ((\mathrm{TESMF} / \mathrm{TGPF}) - 0.4 \times (\mathrm{OFFESMF} / \mathrm{GP})) / 0.6 \times \mathrm{GP}$$

But wait a second; the sharper amongst you may have noticed the numbers I listed for Selanne and Ricci, and the numbers that would result from applying the above formula. Selanne, who is an offensive specialist, would wind up with 1,051 DEFESMF (12.82 per GP), while Ricci, a strong two-way player, would have 924 DEFESMF (11.70 per GP). These results argue that Selanne is better defensively than Ricci, which we know is not the case.

So what's going on? Well, Teemu Selanne is an acknowledged offensive force. This means that his opponents will tend to play their best defensive units against him, which depresses his scoring totals. This, in turn, inflates his DEFESMF. That's nice to know, but how do we adjust for it? There's nothing we can do directly, but we can make an adjustment based on inference.

A team's best offensive players will tend to play the most on the PP. Therefore, we can use a player's PPM per GP as an indication of how much he winds up playing against the best defence of his opponents. First we need to calculate the total PPM played by San Jose forwards (PPMF); this is 2,106. Per GPF, this is 2.17. Selanne played 4.56 PPM per GP, which we will use to give a bonus to his OFFESMF as follows:

$$\mathrm{OFFESMF2} = \mathrm{OFFESMF} + (\mathrm{PPM} - (\mathrm{TPPMF} / \mathrm{TGPF} \times \mathrm{GP})) / 50$$

This adjustment increases the offensive minutes of players who play an above-average amount on the PP, and reduces the offensive minutes of those who play a below-average amount. The formula for DEFESMF then becomes:

$$\mathrm{DEFESMF2} = ((\mathrm{TESMF} / \mathrm{TGPF}) - 0.4 \times (\mathrm{OFFESMF2} / \mathrm{GP})) / 0.6 \times \mathrm{GP}$$

The corresponding formulae for defencemen are (note the difference in the divisor in the OFFESMD2 calculation):

$$\mathrm{OFFESMD} = (\mathrm{OCES} / \mathrm{ESM}) / (\mathrm{TOCESD} / \mathrm{TESMD}) \times (\mathrm{TESMD} / \mathrm{TGPD}) \times \mathrm{GP}$$

$$\mathrm{OFFESMD2} = \mathrm{OFFESMD} + (\mathrm{PPM} - (\mathrm{TPPMD} / \mathrm{TGPD} \times \mathrm{GP})) / 120$$

$$\mathrm{DEFESMD2} = ((\mathrm{TESMD} / \mathrm{TGPD}) - 0.4 \times (\mathrm{OFFESMD2} / \mathrm{GP})) / 0.6 \times \mathrm{GP}$$

That covers a player's PP time. What about his SH time? We make an adjustment for this as well, but for a different reason. Whereas a high amount of PP time indicates a player will be targeted by opposing defences, a high amount of SH time indicates the player is probably used against the best opposing offensive players. While the effect on a player's offence due to this is questionable, it does mean that players who play a lot short-handed will tend to play in high-leverage defensive situations. That is, by playing against the best scorers, the player can have a greater effect upon how many goals his team allows. We therefore adjust a player's DEFESMF2 and DEFESMD2 as follows:

$$\mathrm{DEFESMF3} = \mathrm{DEFESMF2} + (\mathrm{SHM} - (\mathrm{TSHMF} / \mathrm{TGPF} \times \mathrm{GP})) / 50$$

$$\mathrm{DEFESMD3} = \mathrm{DEFESMD2} + (\mathrm{SHM} - (\mathrm{TSHMD} / \mathrm{TGPD} \times \mathrm{GP})) / 120$$

Note that the divisors used here have no real basis; they are part of the artistry of the method. 50 and 120 are used because they produce results that are “about right”.

So we can move on and allocate DPS3EST now, right? Not yet; there are still two more adjustments to make. I did tell you the method was heavily modified.

We can't forget one of our significant assumptions. When deciding how much defensive credit to give goaltenders, we decided that each position on the ice (three forwards, two defenceman and one goalie) should receive equal value. That means that when allocating DPS2 to forwards and defencemen, we must make sure to do it in such a way as to achieve this equality (on the whole).

We know the offensive side of the equation. In 2002/03, NHL forwards scored 5,613 goals, and NHL defencemen scored 917. Forwards had 8,071 assists, while defencemen had 3,046. Using our formula for OC, we find that three forward positions combined for 5,147.09 OC (1,715.70 each), and the two defence positions combined for 1,341.84 OC (670.92 each), for a combined total of 6,488.93. We can therefore express each position's relative offensive contributions as follows:

F1	.264
F2	.264
F3	.264
D1	.104
D2	.104
G	.000

Each position is to be worth .167 of a team's total value of 1.000. Since offence is 40% of hockey, value contributed by offence is as follows:

F1	.106
F2	.106
F3	.106
D1	.041
D2	.041
G	.000

So the value needed from defence is:

F1	.061
F2	.061
F3	.061
D1	.125
D2	.125
G	.167

Which means each position's relative defensive contribution is:

F1	.102
F2	.102
F3	.102
D1	.208
D2	.208
G	.278

This means that when allocating DPS3, we must ensure that, on the whole, each defence position receives 2.04 times as many DPS3 (.208 / .102) as each forward position. So we can just multiply DEFESMD3 by 2.04 to get DEFESMD4, right? Not quite.

We're allocating defence like so: 69.1% to ES, 27.4% to SH and 3.5% to PP. Assuming ES and PP time is played with three forwards and two defencemen, while SH time is played with two of each, each forward position gets 19.09% of defence (13.82% ES, 4.57% SH, 0.70% PP) and each defence position gets 21.37% of defence (13.82% ES, 6.85% SH, 0.70% PP), based solely on ice time.

So we need to transform each defence position's 21.37% into 38.94% (2.04 times 19.09%). But we're only adjusting ES time; so the 17.57% difference (38.94 less 21.37) gets added to the 13.82 from ES, for a total of 31.39. Therefore the ES multiplier is not 2.04, but 2.27 (31.39 / 13.82). Thus:

DEFESMD4 = DEFESMD3 x 2.27

While:

DEFESMF4 = DEFESMF3

And we're still not quite done. To this point, the DPS3EST for each team is being distributed between forwards and defencemen using the same ratio. This is not representative of reality. The relative defensive prowess of each team's forward and defence units is not constant, and players compete only against other who play their position on the team for ice time. But how can we determine what this difference might be?

When a team plays short-handed, they generally play two forwards and two defencemen; 50% of the skaters are defencemen. In other situations, defencemen generally make up only 40% of all skaters (two out of five, occasionally less on the PP). From this we can infer quite reasonably that if a team is better defensively while short-handed than we would expect based on their even-strength play, their defence corps is relatively better defensively than their forwards. Thus a greater proportion of DPS3EST should be attributed to their defencemen than an average team.

For the 2002/03 NHL season, goals per minute on the PP were scored 2.48 times as often as at ES. So, if a team’s goals-per0\-minute rate while short-handed was, say, 2.20 times its ES rate, we would conclude that this team’s defence corps is relatively better defensively than its forward unit.

We could simply multiply each defenceman’s DEFESMD4 by the average rate divided the team rate. But this, in fact, produces very significant variation. It is difficult to justify too great an adjustment, so first we normalize the team rate by averaging it with the league rate. Thus:

DEFESMD5 = DEFESMD4 x ((PPGL / PPML) / (ESGL / ESML)) / ((((PPGL / PPML) / ESGL / ESML)) + ((PPGAT / SHMT) / (ESGAT / ESMT))) / 2)

DEFESMF5 = DEFESMF4

Now we can finally move on to...one final adjustment. This “artistic” adjustment is needed because without it, there would simply be too much variation in DEFESMD5 among a team’s defencemen. Unlike forwards, you simply cannot be an NHL defenceman without being able to play a certain amount of defence. Even a player like Sergei Gonchar, known as an offensive specialist, is far better defensively than many NHL forwards. It is a requirement of the position, unless your name is Paul Coffey. This final adjustment normalizes the defencemens' DEFESMD5 by averaging it with the team average.

DEFESMD6 = (DEFESMD5 + ((sumDEFESM5 / TGPD) x GP)) / 2

DEFESMF6 = DEFESMF5

Where sumDEFESM5 is the sum of a team's defencemens' DEFESM5.

We can now allocate DPS3EST to individual players. This is the final step. It is allocated based on each player's DEFESM6, in proportion to the team total.

The Final Result: Points Allocated

All players now have a number of offensive points allocated to them (equal to OP2), as well as a number of defensive points (equal to DPS3PPT + DPS3SHT + DPS3EST + DPIG if applicable) and a number of points lost to penalties (CPT). The sum of these figures is the player's Total Points Allocated (TPA), which is a reflection of the player's true value.

Appendix: List of Abbreviations Used

AvgDPS2	League avarage defensive points allocated to skaters
AvgG	League average goals for or goals against
AvgMIN	League average goaltender minutes
AvgSA	League average shots against
AvgSPCT	League average save percentage modified by Ryder's shot quality analysis
CP	Cost of individual penalty
CPLG	Cost of penalties in goals
CPLP	Cost of penalties in expected points
CPT	Team cost of penalties
DEFESMD	Individual defenceman's defensive even-strength minutes; also DEFESMD2, 3, 4, 5, 6
DEFESMF	Individual forward's defensive even-strength minutes; also DEFESMF2, 3, 4, 5, 6
DP	Defensive points, team or individual; also DPS2, 3
DPG	Team defensive points allocated to goaltenders
DPIG	Defesive points allocated to individual goaltender
DPS	Team defensive points allocated to skaters
DPS3ES	Ratio of skater defensive points allocated to even-strength situations
DPS3PP	Ratio of skater defensive points allocated to power-play situations
DPS3SH	Ratio of skater defensive points allocated to short-handed situations
DPS3EST	Team skater defensive points allocated to even-strength situations
DPS3PPT	Team skater defensive points allocated to power-play situations
DPS3SHT	Team skater defensive points allocated to short-handed situations
DS	Team defensive synergy
EPTS	Team expected points
ES	Even-strength
ESGAT	Team even-strength goals against
ESGL	League even-strength goals
ESM	Individual even-strength minutes
ESML	League even-strength minutes
ESMT	Team even-strength minutes
GA	Team goals against
GF	Team goals for
GL	League goals scored
GP	Individual games played
GPPT	League goals per expected point
MGF	Team marginal goals for
MGR	Marginal goal rate
MGS	Team marginal goals saved
MGSG	Team marginal goals saved by goaltenders
MGSS	Team marginal goals saved by skaters
MGSGIG	Marginal goals saved by individual goaltender
MIN	Individual goaltender minutes
NPPGL	League non-power-play goals
NPPML	League non-power-play minutes
OC	Individual offensive contribution
OCES	Individual offensice contribution at even-strength
OFFESMD	Individual defenceman's offensive even-strength minutes; also OFFESMD2
OFFESMF	Individual forward's offensive even-strength minutes; also OFFESMF2
OP	Offensive points, team or individual; also OP2
OS	Team offensive synergy
PA	Point allocation
PAR	Points above replacement
PP	Power-play
PPGAT	Team power-play goals against
PPGL	League power-play goals
PPM	Individual power-play minutes
PPML	League power-play minutes
PPMT	Team power-play minutes
PPOL	League power-play opportunities
SCH	League length of schedule (number of games per team)
SH	Short-handed

SHGAT	Team short-handed goals against
SHGL	League short-handed goals
SHM	Individual short-handed minutes
SHMT	Team short-handed minutes
SHM2D	Individual defenceman's modified short-handed minutes
SHM2F	Individual forward's modified short-handed minutes
SPCT	Team save percentage modified by Ryder's shot quality analysis
SRateG	League synergy rate in goals
SRateP	League synergy rate in points
SPCTG	Individual goaltender save percentage modified by Ryder's shot quality analysis
SumDEFESMD5	Sum of a team's defencemen's defensive even-strength minutes
SumMGSIG	Sum of marginal goals saved by individual goaltenders on a team
TESMD	Total even-strength minutes played by a team's defencemen
TESMF	Total even-strength minutes played by a team's forwards
TGPD	Total games played by a team's defencemen
TGPF	Total games played by a team's forwards
TML	League total minutes played
TMS	Number of teams in league
TOCESD	Total even-strength offensive contribution of a team's defencemen
TOCESF	Total even-strength offensive contribution of a team's forwards
TPA	Individual total points allocated
TPPMD	Total power-play minutes of a team's defencemen
TPPMF	Total power-play minutes of a team's forwards
TSHMD	Total short-handed minutes of a team's defencemen
TSHMF	Total short-handed minutes of a team's forwards
TSH	Team times short-handed

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